Written Exam for the B.Sc. in Economics 2010-II

Micro 3

Final Exam

August 23rd 2010

(2-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

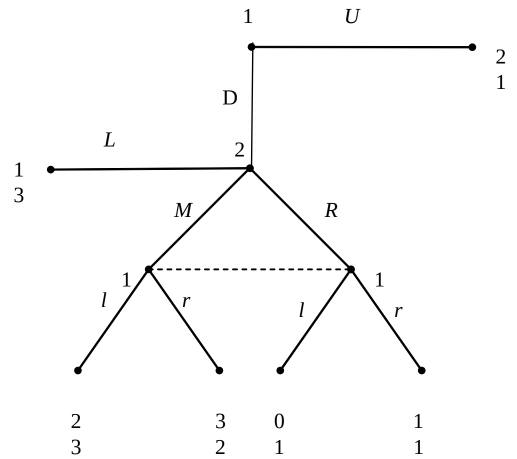
If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

PLEASE ANSWER ALL QUESTIONS BELOW. PLEASE EXPLAIN YOUR ANSWERS.

1. (a) Find *all* Nash equilibria in the following game

	L	R
T	$2,3$	$0,2$
B	$1,0$	$4,7$

   (b) Consider the extensive-form game below



      i. Describe the strategy sets of all the players and count the number of subgames.
      ii. Find all its (pure-strategy) subgame perfect Nash equilibria.

   (c) Consider the following game between two players

	C	D
C	$2,2$	$-1,3$
D	$3,-1$	$1,1$

      i. Solve this game by iterated elimination of strictly dominated strategies
      ii. Assume that this game is repeated infinitely many times $t = 1, 2, 3, ..., \infty$, and both players maximize the sum of their own future discounted payoffs. Assume further that both players have a common discount factor $\delta = 2/3$. Propose a "grim trigger strategy" SPNE of this game, such that outcome (C,C) is played in each period *along the game path*. Demonstrate that your proposal is indeed supported as an SPNE (i.e. that no player wants to deviate from the proposed rule).
      iii. Consider the following statement: "Assume that a player has a *strictly dominant* strategy s in a stage game G (i. e. the strategy that makes him better off than any of his other strategy, no matter what the other players do). Now assume that G is repeated infinitely many times, and players maximize the sum of their future discounted payoffs. Then playing s in each period of $G(\infty)$ would also give this player the highest payoff". True or false? Explain your answer.

2. Two neighboring countries are contributing to the monitoring of the activity of a nearby volcano. They simultaneously and non-cooperatively decide on level of the technology they want to use at their respective geological stations. If country 1 chooses level $x_1 \in [0, 1]$ and country 2 chooses level $x \in [0, 1]$, then the probability that they will detect an upcoming volcano eruption is

$$\frac{(x_1 + 1)(x_2 + 1)}{4}.$$

The cost of obtaining a technology level x_1 in country 1 is $\frac{(x_1)^2}{4}$, and is known to both countries. The cost of obtaining a technology level x_2 in country 2 is $\frac{t(x_2)^2}{4}$, where $t > 0$. Each country maximizes the probability of detecting the eruption less the technology cost. That is, the payoff to country 1 is

$$u_1(x_1, x_2) = \frac{(x_1 + 1)(x_2 + 1)}{4} - \frac{(x_1)^2}{4},$$

and the payoff to country 2 is

$$u_2(x_1, x_2) = \frac{(x_1 + 1)(x_2 + 1)}{4} - \frac{t(x_2)^2}{4}.$$

(a) Assume that $t = 1$, and it is known to both countries. Derive the best response functions of both countries and find the technology levels chosen by the countries in NE of this game.

(b) Assume instead that $t = 2$, i.e. that the technology in country 2 is more costly. Again, both countries know that $t = 2$. Derive the best response functions of both countries and find the technology levels chosen by the countries in NE. Is country 1 choosing a higher or a lower level of technology in (b), as compared to (a)? Why? Provide some intuition.

(c) Now assume that t, the exact value of the technology cost parameter of country 2, is only known to country 2. Country 1 only knows that t can be either $t_L = 1$ with probability 1/3, or $t_H = 2$ with probability 2/3.

i. What are the types of both players in this static game of incomplete information? What are the strategies of the players?

ii. What is the best response of country 2 with the technology cost parameter $t_L = 1$, $x_2^L(x_1)$? Of country 2 with $t_H = 2$, $x_2^L(x_1)$?

iii. What is the best response of country 1, $x_1(x_2^L, x_2^H)$?

iv. Find the Bayes-Nash equilibrium of this game.

3. Anna and Bjorn are playing the alternating-offer bargaining game in order to divide 1 unit of (infinitely divisible) output. Their preferences are given just by the amount of output they get. Anna discounts future at the rate $\delta_A \in (0, 1)$, and Bjorn discounts future at the rate $\delta_B \in (0, 1)$.

(a) Assume that the game lasts for 3 periods. In period 1 Anna makes an offer on how to split the output: s_1 to herself, $1 - s_1$ to Bjorn. Bjorn observes the offer and decides whether to accept or reject it. If the offer is accepted, the game is over and the players get the accepted offers. Otherwise the game proceeds to period 2. In period 2 Bjorn makes an offer: s_2 to Anna and $1 - s_2$ to Bjorn himself. Anna observes the offer and chooses to accept or to reject it. Again, if the offer is accepted, the game is over, otherwise it proceeds to period 3. In period 3 Anna receives (exogenously determined) share s of output and Bjorn receives share $1 - s$.

What are the payoffs that Anna and Bjorn get in a SPNE of this game? (you may assume that whenever a player is indifferent between accepting and rejecting the offer, she or he accepts.)

(b) Now assume that the alternating-offer bargaining game between Anna and Bjorn runs for potentially infinite number of periods. Again, Anna is the one who makes the first offer. Show that the payoff of Anna in the SPNE of this game is given by

$$s_A = \frac{1 - \delta_B}{1 - \delta_A \delta_B},$$

and the payoff of Bjorn is given by

$$s_B = \frac{\delta_B (1 - \delta_A)}{1 - \delta_A \delta_B}.$$

(HINT: we have argued in the lectures that for the infinite-period alternating offer game the subgame that starts in period 3 looks exactly like the subgame that starts in period 1. What does it imply for the payoff that Anna receives in the game that starts in period 3, and in the game that starts in period 1?)

How does the payoff of Anna depend on her own discount factor? On the discount factor of Bjorn? Provide intuition behind your answer.